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TRADITIONAL PATTERNS OF EASTER EGGS IN THE CARPATHIAN BASIN AND SPHERICAL SYMMETRIES

Eleonóra STETTNER, Rózsa NIENHAUS

Abstract: In this article we are analyzing which spherical symmetry groups can be found and which ones cannot among the eggs with wax-resist patterns most frequently used in the Carpathian Basin. We are also trying to answer the question why exactly these symmetry decorations can be found. Before that, we are providing a review on the history of decorating eggs and of the above-mentioned decoration technique. By publishing this article, our unconcealed aim is also to make this tradition and the related precious patterns more widely known. The author of the second chapter of the article is Rózsa Nienhaus, the founder of the Egg Art Museum.

Key words: spherical symmetry, symmetry group, patterns of easter eggs

1. Introduction

The concept of symmetry is inevitable in the relationship between mathematics and art. While preparing for my classes on Mathematics and Art, or for the activities of the Experience Workshop, I was surprised by the richness of symmetries applied in the old, several hundred or sometimes even several thousand-year-old works of folk art. The study of the works and life of the Dutch painter, M. C. Escher leads one inevitably to the patterns of the Alhambra tiles. Surprisingly, all 17 wallpaper groups can be found here (Pérez-Gómez, 1987). The patterns of the Alhambra were created in the thirteenth-fifteenth centuries A.D. Mathematicians provided a complete classification of the plane crystal groups at the turn of the 19th and 20th centuries. In mathematics it often happens that several mathematicians prove a theorem, independently of one another, almost at the same time. The same happened here. 'In the E^2 Eucledean plane there are 17 crystal groups. Their first complete list was published by Fjodorov (1890) along with the 219+11 spatial crystal groups, which were discovered at the same time by Schoenflies (1891). Fricke and Klein also dealt with plane crystal groups in 1897, and then György Pólya as well, in 1924. Nowacki described crystal groups in an abstract way.' (Stettner, 2004).

All 17 groups can be found on authentic Hungarian folk art embroidery works, which were not created in the last 200 years, either (Hargittai & Lengyel, 1985). Moreover, Szaniszló Bérczi draws attention to the fact that the 7 frieze symmetries can also be found in Hungarian ornamental art from the time of the Hungarian conquest (Bérczi, 2010). All these frieze symmetries can be found on Hungarian folk art embroidery (Hargittai & Lengyel, 1984), what is more, all of them come up if we only examine embroidery made using a special technique, cross-stitch.

A good example of spherical symmetries is the one thousand-year-old Japanese technique of temari. Originally it was a simple toy ball made for children, but nowadays anyone can learn how to create the amazing patterns with the help of several Youtube videos (http://temari.com/). The article written by Carolyn Yackel and presented at the greatest mathematical art world conference also draws attention to the relationship between art and the mathematical content of temari balls (Yackel, 2011).

I first met egg decorating patterns put in a systematic form in the article of Erzsébet Györgyi (Györgyi, 1974). Thanks to my relationship with the founder of the Egg Art Museum (Zengővárkony), among several thousands of decorated eggs I had the opportunity to meet various decorating techniques, and to examine and analyze them thoroughly related to our topic. I found that eggs that were made using

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the wax-resist technique have the richest symmetry. Thus, the present article studies these ones, and illustrates occurring spherical symmetries using items from the photo collection of the museum.

’The vast majority of egg decorations (considered archaic by character) were created with this process, and we have reason to believe that this technique was common among peasant hands during centuries before the one hundred years we are able to examine’ (Györgyi, 1974).

Eggs are topologically homeomorphic with the sphere, which makes it possible to compare egg patterns and spherical symmetries. The classification of patterns follows the system of spherical symmetry groups.

Finally, as with all works of art made by humans, we have to mention the concept of dissymmetry[1]. The pattern cannot be completely, perfectly symmetrical, this is partly due to the bearer of the pattern, the egg, and partly to the nature of human creation.

2. From the Bare Egg to the Decorated One

Eggs, being the bearer, or in particular cases the source of life, have a special cultural-historical significance. The central element of various creation myths is the egg (ancient egg) from whose division the elements of the universe can be originated. In European context, as a creation myth, the Kalevala has to be mentioned (Makoldiné Papp, 1994), (Tészabó, 1990).

The decoration of eggs is an ancient activity. Different ethnic groups attributed magic power to the egg. Its decoration was probably supposed to enhance this magic power. They were also placed in graves as cultic objects. According to the literary reference that we know, a South-African ostrich egg find is 60,000 years old (https://www.cam.ac.uk/research/news/egg-cetera-6-hunting-for-the-worlds-oldest-decorated-eggs). In Hungary, Avar egg fragments with scratched lines on them were unearthed during the excavation led by Ferenc Móra (Móra, 1932). Other examples are mentioned, among others, by Tészabó Júlia: inwrought stone egg from Egypt around 500 B.C. (Tészabó, 1990), Sándor Beluleszkó:’ The custom that close friends and acquaintances give each other decorated eggs as a present, is ancient and extremely widespread. It is rooted in the spring ceremonies of the pagan age, and we can find it among most nations of antiquity, e.g. in China it has been known for more than 2,000 years. It was a symbolic expression of the joy that people felt at the beginning of spring over the rebirth and resurrection of Nature. It was taken over by Christianity as well, like most customs which were based on the traditions of peoples who accepted Christian doctrines. Its symbolic meaning then changed as ’it became the symbol of Christ’s Resurrection’ (Beluleszko, 1905).

These sources show the ancient origins of egg decoration and its prevalence all over the world. And the last quotation leads us to the Easter of our time. The discussion of traditions and symbology related to Easter, eggs and their decoration is not a topic of the present article for reasons of space. The comparison of the spherical symmetries with the patterns of the decorated eggs is done by presenting eggs decorated with patterns from the Transylvanian regions of the Carpathian Basin.

Zengővárkony is a small village, with rich folk art traditions, 17 km from Pécs, the centre of the county, and situated at the foot of the Eastern Mecsek, in picturesque surroundings. Since 16th April, 2000 there has been a permanent exhibition here, the Museum Arts on Egg, which displays only decorated eggs (in Figure 1). The museum has a collection of nearly 5,000 eggs, of which visitors can see approximately 2,000 exhibits in the glass-cases, mostly waxed, painted and scratched eggs. The appliquéd eggs are especially interesting, here the decoration is formed by fixing different materials onto the surface of the egg. The aims of the museum are to show the folk traditions of decorating eggs and to collect and present its relevant objects. The egg patterns shown on the illustrations of the article were created using the wax-resist process.

[1] Dissymmetry is a slight injury to symmetry. The phenomenon, law or pattern preserves symmetry in its main lines, but symmetry does not necessarily apply in its details.

One of the most frequently used decoration methods is drawing with wax (in Figure 2). With the help of a special device we draw patterns on the surface of the egg using melted wax. We can color those parts that are left out by putting the egg into paint. If after drying the wax remains on the surface, we get a wax appliquéd egg, and if we wipe it off, we get a wax-resist egg. By repeating these simple steps with ’artistic expertise’ we can create colorful compositions.

Easter eggs were decorated on either Holy Thursday or Good Friday, according to the traditions of the particular region. Like in every creative process shaping material, it also starts with getting to know the material, that is, the bare egg. The decorators took it in their hand, and holding and caressing it they decided what patterns to put on it, and most of the time the pattern was being formed ’on the go’. An exception to this is the pattern of the lost traveller or mistaken traveller common in Transyvania, which was considered to be the most complicated one, and thus it was drawn first, in silence, with full focus of the women’s attention. The outstretched drawing of the pattern is a labyrinth. Drawing it on an egg is not an easy task technically either, one needs to have an unerring touch and a good sense of proportion to be able to do it.

Figure 1. *The Egg Art Museum, Zengővárkony*

Figure 2. *Egg decoration with a special device*

3. Geometric Surface Division Network on the Surface of the Egg

The organization of the treasury of egg decoration has been completed from several aspects, a very thorough work is the study of Erzsébet Györgyi (Györgyi, 1974). The present article approaches systematization from the aspect of mathematics, according to spherical symmetry groups. As far as we know, the classification of wax-resist egg patterns by this criterion has not been done before.

’The decoration of Easter eggs is shaped by the following factors: the decorating technique, egg surface distribution, applied decorations, possible inscriptions, the placement of the latter two on the divided or undivided surface of the egg, and coloring.’ (Györgyi, 1974). From the aspect of symmetries, we are interested in surface division, because it can affect the occurrence of symmetry groups.

The surface dividing lines can be longitudinal bisectors, cross-side bisectors or oblique bisector pairs (Figure 3 and Figure 4). In Figure 3 on the left longitudinal bisectors can be seen from side view, and in Figure 4 they can be seen from top view. Lines marked with ’a’ are the so-called first-order bisectors, and the ones marked with ’b’ are the second-order bisectors. The middle image in Figure 3 illustrates the cross-side bisectors; ’b’ indicates the cross-side bisector, ’a’ shows the upper bisector and ’c’ indicates the lower one. And in the image on the right in Figure 3 oblique bisector pairs can be seen.

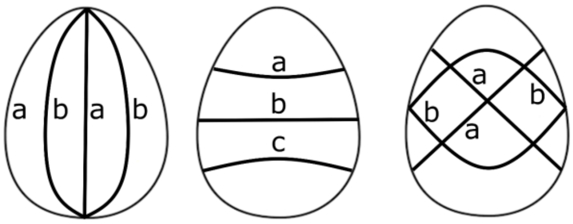

Figure 3. *Surface dividing lines, side view*

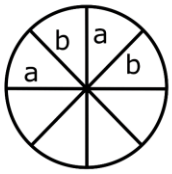

Figure 4. *Surface dividing lines, top view*

It had puzzled me for a long time that three-fold symmetries (both reflections and rotations) appear on the eggs, but five-fold ones do not. If on the surface of the egg only one type of surface dividing lines (longitudinal, cross-side bisectors or oblique bisector pairs) is drawn, no three-fold symmetries could appear, as the lines always divide the egg surface into 2 power portions. It is also important to mention here that egg decorators' desire to achieve symmetry is shown by the fact that if they draw a cross-side bisector or oblique bisector in the upper half of the egg, it also appears in its lower half. But if surface dividing lines of different types can appear together on the egg surface, then three-fold symmetries can appear too, however, five-fold ones cannot. Figure 5 illustrates dividing lines of different types on egg surface in two common combinations. In the drawing on the left we can see clearly that three surface dividing lines meet in the points marked with red. Thus, at these points three-fold rotation or reflection symmetries can occur. This surface division created 48 triangles, so if a pattern follows the system of Figure 5, there may be a 48-fold symmetry group. If the pattern combines two or four adjacent triangles, the order of the group can be 24 and 12 as well. A different division may also lead to three-fold symmetry. In the middle image we drew the longitudinal and cross-side bisectors. Thus, on the surface of the egg 16 triangles and 16 rectangles were formed. Due to the tapered shape of the egg at the two ends, we get a relatively even distribution if each of the sixteen rectangles is divided into triangles. Thus, 16 + 2 • 16 = 48 triangles will be generated and here as well there will be points where 6 triangles meet. Filling these with alternating colurs or patterns three-fold symmetries can occur here, too In the right image of Figure 5 an egg painted this way can be seen. Certainly, patterns containing three-fold symmetries may not only consist of triangles; any pattern that follows the illustrated systems can be similar.

Figure 5. *Dividing lines of different types on egg surface in two common combinations*

4. Egg Patterns and Spherical Symmetry Groups

Spherical symmetry groups have an infinite number of elements, which can be divided into 14 groups/families, as we can see in Table 1. There are symmetry groups, for which we can find several egg patterns, as they were extremely popular with egg decorators, such as groups 22N, *NN, *22N, where cases N = 2 and N = 4 are illustrated by a lot of examples. The most difficult was to find a pattern for the last mixed group – which contains rotation and glide reflection – I could find only one in the literature (Györgyi, 1974). Obviously, this pattern only corresponds to the symmetries of the given group if we disregard the random dots (in Figure 15).

We can analyze the patterns by focusing only on the lines of the drawing and the forms, and disregard the different filling types of the domains (domains of different colours, domains with dots and hatches). The example for this is the pattern on Figure 13, following the symmetry of group *332. The pattern of the egg on the left completely corresponds to the group – considering the fillings as well, while the patterns of the one in the middle and on the right correspond only if we disregard the colorings. I would also like to highlight the interesing egg pattern belonging to group 332, shown in Figure 8, which recalls Escher's plane or sphere covering patterns. Here the decorator of the egg drew congruent figures on the egg, and the pattern consists of 4 dotted and 4 plain three-fold rotational symmetrical domains. The fundamental domain is the combination of one-third dotted and one-third plain domains, so we get a twelve-fold group (3*8=24, 24/2=12). If we did not distinguish between plain and dotted domains, there would be 432 groups, the fundamental domain of which would be one-third of the 'beautiful three-armed shape' and the order of the resulting group would be 24.

We can notice that the only patterns missing from groups containing exclusively rotations or reflections are the ones containing five-fold symmetry. There are no patterns for two of the mixed groups but, as I have mentioned before, I also had difficulties in finding a not entirely adequate example for the last one.

Table 1. *A review of spherical symmetry groups*

The sign of the spherical group	The order of the spherical group	The sign of the egg's symmetry group	The order of the egg's symmetry group	The image of the egg
Groups containing only rotation symmetry				
NN	N	44	4	Figure 6.
22N	2N	226	12	Figure 7.
332	12	332	12	Figure 8.
432	24	432	24	Figure 9.
532	60	—	—	—
Symmetry groups containing only reflection				
*NN	2N	*22	4N	Figure 10.
*22N	4N	*224	16	Figure 11.
*432	48	*432	48	Figure 12.
*532	120	—	—	—
*332	24	*332	24	Figure 13.
Mixed symmetry groups containing reflection, rotation, or glide reflection as well				
3*2	24	—	—	—
N*	2N	—	—	—
2*N	4N	2*3	12	Figure 14.
Nx	2N	4x	8	Figure 15.

Figure 6.

Figure 7.

Figure 8.

Figure 9.

Figure 10.

Figure 11.

Figure 12.

Figure 13.

Figure 14.

Figure 15.

Conclusions

The symmetry inherent in Easter egg patterns draws attention to the relationship between mathematics and art, or more specifically, between mathematics and folk art. After reviewing several different sample collections and consulting egg decorators, we succeeded in finding patterns for most symmetry groups. (If anyone among our readers knows about a missing pattern, and sends it to us, we will appreciate it.) One of the authors is a founding member and research coordinator of the Experience Workshop, so we consider it important to support one of its main endeavors, the teaching of STEAM (Science, Technology, Engineering, Arts, Mathematics). One of our plans is to have an Experience Workshop event for children at the Egg Art Museum in Zengővárkony during the week before the Easter holiday, where they can get acquainted not only with the technique of egg decoration but also with the symmetry of patterns and thus, during a playful activity they can acquire mathematical contents in an easy way.

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Authors

Eleonóra STETTNER, Kaposvár University, Kaposvár (Hungary), email: stettner.eleonora@gmail.com

Rózsa NIENHAUS, Museum Arts on Egg, Zengővárkony (Hungary), email: nienhau@uni-muenster.de, tojas@museum.hu